Definitions and Theorems:

**Congruent Triangles:**

- **SSS ≅ SSS**
- **SAS ≅ SAS**
- **ASA ≅ ASA**
- **AAS ≅ AAS**
- **HL ≅ HL**

**Definitions:**
1. Two lines form right angles
2. An angle bisector divides an angle into two congrucent angles
3. Complementary angles are two angles whose sum is right angles
4. A midpoint divides a segment into two congruent segments
5. Two angles that form a linear pair are supplementary
6. A bisector divides segments into two congruent segments

**Postulates:**
1. Reflexive: \( AB = AB \) or \( \angle A = \angle A \)
2. Transitive: if \( AB = BC \) or if \( \angle 1 = \angle 2 \)
   \[ BC = CD \] or \[ \angle 2 = \angle 3 \]
   Then \( AB = CD \) then \( \angle 1 = \angle 3 \)
3. Addition - congruent segments + congruent segment are congruent
   congruent angles + congruent angles are congruent
4. Subtraction - congruent segments - congruent segments are congruent
   Congruent angles - congruent angles are congruent

Theorems:
1. Right angles are congruent
2. Straight angles are congruent
3. Complements of the same angle are congruent
4. Supplements of the same angles are congruent
5. Complements of congruent angles are congruent
6. Supplements of congruent angles are congruent
7. Vertical angles are congruent

Examples:
1. Given: Parallelogram $ABCD$ with diagonal $\overline{AC}$ drawn

   \[ \triangle ABC \cong \triangle CDA \]

2. In the diagram below of circle $O$, tangent $\overline{EC}$ is drawn to diameter $\overline{AC}$. Chord $\overline{BC}$ is parallel to secant $\overline{ADE}$, and chord $\overline{AB}$ is drawn.

   \[ \frac{BC}{CA} = \frac{AB}{EC} \]
3. Isosceles trapezoid $ABCD$ has bases $\overline{DC}$ and $\overline{AB}$ with nonparallel legs $\overline{AD}$ and $\overline{BC}$. Segments $AE, BE, CE,$ and $DE$ are drawn in trapezoid $ABCD$ such that $\angle CDE = \angle DCE$, $\overline{AE} \perp \overline{DE}$, and $\overline{BE} \perp \overline{CE}$.

![Diagram of trapezoid with segments drawn](image)

Prove $\triangle ADE \cong \triangle BCE$ and prove $\triangle AEB$ is an isosceles triangle.

4. Kelly is completing a proof based on the figure below.

![Diagram of triangle with additional segments](image)

She was given that $\angle A = \angle EDF$, and has already proven $\overline{AB} \cong \overline{DE}$. Which pair of corresponding parts and triangle congruence method would not prove $\triangle ABC \cong \triangle DEF$?

- (1) $\overline{AC} \cong \overline{DF}$ and SAS
- (2) $\overline{BC} \cong \overline{EF}$ and SAS
- (3) $\angle C \cong \angle F$ and AAS
- (4) $\angle CBA \cong \angle FED$ and ASA
5. In quadrilateral \textit{BLUE} shown below, $\overline{BE} \cong \overline{UL}$.

Which information would be sufficient to prove quadrilateral \textit{BLUE} is a parallelogram?

(1) $\overline{BL} \parallel \overline{EU}$
(2) $\overline{LU} \parallel \overline{BE}$
(3) $\overline{BE} \cong \overline{BL}$
(4) $\overline{LU} \cong \overline{EU}$

b. Given: $\overline{RS}$ and $\overline{TV}$ bisect each other at point \( X \)

$\overline{TR}$ and $\overline{SV}$ are drawn

Prove: $\overline{TR} \parallel \overline{SV}$
7. In the diagram below, if $\triangle ABE \cong \triangle CDF$ and $\overline{AEFC}$ is drawn, then it could be proven that quadrilateral $ABCD$ is a

- (1) square
- (2) rhombus
- (3) rectangle
- (4) parallelogram

8. In the diagram below, $\overline{GI}$ is parallel to $\overline{NT}$, and $\overline{IN}$ intersects $\overline{GT}$ at $A$.

Prove: $\triangle GIA \sim \triangle TNA$
9. Given: Circle $O$, chords $\overline{AB}$ and $\overline{CD}$ intersect at $E$

![Circle with intersecting chords](image)

Theorem: If two chords intersect in a circle, the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.

Prove this theorem by proving $AE \cdot EB = CE \cdot ED$.

10. Given: Parallelogram $ABCD$, $\overline{EFG}$, and diagonal $\overline{DFB}$

![Parallelogram with diagonal](image)

Prove: $\triangle DEF \sim \triangle BGF$

11. Given: Quadrilateral $ABCD$ with diagonals $\overline{AC}$ and $\overline{BD}$ that bisect each other, and $\angle 1 = \angle 2$

![Quadrilateral with bisecting diagonals](image)

Prove: $\triangle ACD$ is an isosceles triangle and $\triangle AEB$ is a right triangle
Topic 2: SOH, CAH, TOA, Law of Sines, Law of Cosines, and Area

Formulas

Law of Sines
\[ \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \]

Law of Cosines
\[ c^2 = a^2 + b^2 - 2ab \cos C \]

Area of Triangle
\[ k = \frac{1}{2} ab \sin C \]

SOH CAH TOA
\[ \sin \theta = \frac{O}{H} \quad \cos \theta = \frac{A}{H} \quad \tan \theta = \frac{O}{A} \]

Examples:

1. Given: Right triangle ABC with right angle at C

   If \( \sin A \) increases, does \( \cos B \) increase or decrease? Explain why.

2. Bob places an 18-foot ladder 6 feet from the base of his house and leans it up against the side of his house. Find, to the nearest degree, the measure of the angle the bottom of the ladder makes with the ground.
3. A man was parasailing above a lake at an angle of elevation of 32° from a boat, as modeled in the diagram below.

If 129.5 meters of cable connected the boat to the parasail, approximately how many meters above the lake was the man?

(1) 68.6
(2) 80.9
(3) 109.8
(4) 244.4

4. As shown in the diagram below, an island (I) is due north of a marina (M). A boat house (H) is 4.5 miles due west of the marina. From the boat house, the island is located at an angle of 54° from the marina.

Determine and state, to the nearest tenth of a mile, the distance from the boat house (H) to the island (I).

Determine and state, to the nearest tenth of a mile, the distance from the island (I) to the marina (M).
5. A ladder 20 feet long leans against a building, forming an angle of 71° with the level ground. To the nearest foot, how high up the wall of the building does the ladder touch the building?

(1) 15  (3) 18
(2) 16  (4) 19

6. Freda, who is training to use a radar system, detects an airplane flying at a constant speed and heading in a straight line to pass directly over her location. She sees the airplane at an angle of elevation of 15° and notes that it is maintaining a constant altitude of 6250 feet. One minute later, she sees the airplane at an angle of elevation of 52°. How far has the airplane traveled, to the nearest foot?

Determine and state the speed of the airplane, to the nearest mile per hour.

7. In the diagram of right triangle \( \triangle ADE \) below, \( BC \parallel DE \).

Which ratio is always equivalent to the sine of \( \angle A \)?

(1) \( \frac{AD}{DE} \)  (3) \( \frac{BC}{AB} \)
(2) \( \frac{AE}{AD} \)  (4) \( \frac{AB}{AC} \)
9. When instructed to find the length of $\overline{HJ}$ in right triangle $HJG$, Alex wrote the equation 
$\sin 28^\circ = \frac{HJ}{20}$ while Marlene wrote $\cos 62^\circ = \frac{HJ}{20}$. Are both students' equations correct? Explain why.

\[ \begin{align*} 
\text{H} & \quad \text{20} \\
\text{G} & \quad \text{28°} \\
\text{J} & \quad \text{J} \\
\end{align*} \]

10. In $\triangle ABC$, where $\angle C$ is a right angle, $\cos A = \frac{\sqrt{21}}{5}$. What is $\sin B$?

(1) $\frac{\sqrt{21}}{5}$ 
(2) $\frac{\sqrt{21}}{2}$ 
(3) $\frac{2}{5}$ 
(4) $\frac{5}{\sqrt{21}}$

11. In the diagram below, a window of a house is 15 feet above the ground. A ladder is placed against the house with its base at an angle of 75° with the ground. Determine and state the length of the ladder to the nearest tenth of a foot.

\[ \begin{align*} 
\text{15 ft} & \quad \text{75°} \\
\end{align*} \]
2. As modeled below, a movie is projected onto a large outdoor screen. The bottom of the 60-foot-tall screen is 12 feet off the ground. The projector sits on the ground at a horizontal distance of 75 feet from the screen.

Determine and state, to the nearest tenth of a degree, the measure of $\theta$, the projection angle.

3. As shown in the diagram below, the angle of elevation from a point on the ground to the top of the tree is $34^\circ$.

If the point is 20 feet from the base of the tree, what is the height of the tree, to the nearest tenth of a foot?

(1) 29.7 (3) 13.5
(2) 16.6 (4) 11.2
The diagram below shows a ramp connecting the ground to a loading platform 4.5 feet above the ground. The ramp measures 11.75 feet from the ground to the top of the loading platform.

Determine and state, to the nearest degree, the angle of elevation formed by the ramp and the ground.

5. Which expression is always equivalent to \( \sin x \) when \( 0^\circ < x < 90^\circ \)?

   (1) \( \cos (90^\circ - x) \)  
   (2) \( \cos (45^\circ - x) \)  
   (3) \( \cos (2x) \)  
   (4) \( \cos x \)

6. As shown in the diagram below, a ship is heading directly toward a lighthouse whose beacon is 125 feet above sea level. At the first sighting, point A, the angle of elevation from the ship to the light was 7°. A short time later, at point D, the angle of elevation was 16°.

   To the nearest foot, determine and state how far the ship traveled from point A to point D.
Topic 3: Circles

Definitions and Theorems:

An **inscribed angle** is one whose vertex lies on the circle and whose rays both intersect the circle.

An **inscribed angle** is equal to one half the intercepted arc.

A **central angle** is one whose vertex lies on the center of the circle and whose rays both intersect the circle.

A **central angle** is equal to the intercepted arc.

The **angles formed by intersecting chords** is given by one half the sum of the measures of the intercepted arc.

A **tangent line** touches a circle in exactly one spot.

A **secant line** is a line that intersects a circle twice.

The **angle formed by tangents and secants** is given by one half the difference of the measures of the far arc and near arc.

The **angle formed by a tangent and a chord** is equal to half of the intercepted arc.

**Secant Segment Length Theorem**

If two secants are drawn from a common exterior point, then the product of the length of one secant with the length of its external segment is equal to the product of the length of the other secant with the length of its external segment.

**Secant/Tangent Segment Length Theorem**

If a secant and tangent are drawn from a common exterior point, then the product of the length of the secant with the length of its external segment is equal to the square of the length of the tangent.

**Intersecting Chords and Segments**

If two chords intersect within a circle, then the product of the partitioned segments of one chord equals the product of the partitioned segments of the other chord.
Examples:

1. In circle $M$ below, diameter $\overline{AC}$, chords $\overline{AB}$ and $\overline{BC}$, and radius $\overline{MB}$ are drawn.

![Circle with chords and diameter](image)

Which statement is not true?

(1) $\triangle ABC$ is a right triangle. (2) $\triangle ABM$ is isosceles. (3) $m\overline{BC} = m\angle BMC$ (4) $m\overline{AB} = \frac{1}{2}m\angle ACB$

2. In the diagram below of circle $O$, chord $\overline{DF}$ bisects chord $\overline{BC}$ at $E$.

![Circle with chord bisected](image)

If $BC = 12$ and $FE$ is 5 more than $DE$, then $FE$ is

(1) 13 (2) 9 (3) 6 (4) 4
3. Quadrilateral $ABCD$ is inscribed in circle $O$, as shown below.

If $\angle A = 80^\circ$, $\angle B = 75^\circ$, $\angle C = (y + 30)^\circ$, and $\angle D = (x - 10)^\circ$, which statement is true?

1. $x = 85$ and $y = 50$
2. $x = 90$ and $y = 45$
3. $x = 110$ and $y = 75$
4. $x = 115$ and $y = 70$

4. In the diagram below of circle $O$, chord $CD$ is parallel to diameter $AOB$ and $m\overline{CD} = 130$.

What is $m\overline{AC}$?

1. 25
2. 50
3. 65
4. 115
5. In the diagram shown below, $\overline{PA}$ is tangent to circle $T$ at $A$, and secant $PBC$ is drawn where point $B$ is on circle $T$.

![Diagram](image)

If $PB = 3$ and $BC = 15$, what is the length of $\overline{PA}$?

(1) $3\sqrt{5}$  (3) 3
(2) $3\sqrt{6}$  (4) 9

6. In the diagram below, $m\overarc{ABC} = 268^\circ$.

![Diagram](image)

What is the number of degrees in the measure of $\angle ABC$?

(1) $134^\circ$  (3) $68^\circ$
(2) $92^\circ$  (4) $46^\circ$
7. In the diagram below, tangent $DA$ and secant $DBC$ are drawn to circle $O$ from external point $D$, such that $AC = BC$.

If $m\overline{BC} = 152^\circ$, determine and state $m\angle D$.

5. In the diagram below of circle $O$, $OB$ and $OC$ are radii, and chords $AB$, $BC$, and $AC$ are drawn.

Which statement must always be true?

(1) $\angle BAC \cong \angle BOC$
(2) $m\angle BAC = \frac{1}{2} m\angle BOC$
(3) $\triangle BAC$ and $\triangle BOC$ are isosceles.
(4) The area of $\triangle BAC$ is twice the area of $\triangle BOC$. 
1. In the diagram below, $\overline{DC}$, $\overline{AC}$, $\overline{DOB}$, $\overline{CB}$, and $\overline{AB}$ are chords of circle $O$, $\overline{FDE}$ is tangent at point $D$, and radius $AO$ is drawn. Sam decides to apply this theorem to the diagram: "An angle inscribed in a semi-circle is a right angle."

Which angle is Sam referring to?

(1) $\angle AOB$  
(2) $\angle BAC$  
(3) $\angle DCD$  
(4) $\angle FDB$

2. In the diagram below of circle $O$ with diameter $\overline{BC}$ and radius $\overline{OA}$, chord $\overline{DC}$ is parallel to chord $\overline{BA}$.

If $m\angle BCD = 30^\circ$, determine and state $m\angle AOB$. 
In the diagram below, quadrilateral $ABCD$ is inscribed in circle $P$.

What is $m\angle ADC$?

1. $70^\circ$
2. $72^\circ$
3. $108^\circ$
4. $110^\circ$

In the diagram of circle $A$ shown below, chords $CD$ and $EF$ intersect at $G$, and chords $CE$ and $FD$ are drawn.

Which statement is not always true?

1. $CG \equiv FG$
2. $\angle CEG \equiv \angle FDG$
3. $\frac{CE}{EG} = \frac{FD}{DG}$
4. $\triangle CEG \sim \triangle FDG$
B. In circle $O$ shown below, diameter $AC$ is perpendicular to $CD$ at point $C$, and chords $AB$, $BC$, $AE$, and $CE$ are drawn.

Which statement is not always true?

(1) $\angle ACB \cong \angle BCD$  
(2) $\angle ABC \cong \angle ACD$  
(3) $\angle BAC \cong \angle DCB$  
(4) $\angle CBA \cong \angle AEC$
Topic 4: Rigid Motion

Definitions:

**Transformations in the Plane**

A **transformation**, $F$, is a function (or rule) that for every point, $P$, in the plane as its input gives or assigns another, single point in the plane, $F(P)$, as its output.

**Rotations**

We can rotate a point $A$ in the plane by an angle of $\theta$ around another point $B$ by constructing segment $A'B$ such that $A'B = AB$ and $m \angle A'BA = \theta$. We say:

$$R_{B,\theta}(A) = A'$$

**Properties of Rotations**

(And All Other Rigid Motions)

1. Rotations transform lines into lines, segments into segments, and rays into rays.
2. Rotations preserve the distance between points and hence the length of line segments.
3. Rotations preserve angles between lines, rays, and segments.

**Geometric Fact:** A line which is rotated 180° about a point not on the line will always result in another line that is parallel to the original. A line which is rotated 180° about a point that is on the line will simply produce the same line.

**Reflections**

We can reflect a point $A$ in the line $m$ to produce $A'$, i.e. $r_m(A) = A'$, by using the following:

1. If $A$ does not lie on $m$ then locate point $A'$ on the other side of $m$ such that segment $AA'$ satisfies:
   a. $AA' \perp m$ and 
   b. $AA'$ intersects line $m$ at its midpoint

2. If $A$ lies on $m$ then its reflection is itself, i.e. $r_m(A) = A$

**Translation Properties**

1. Map lines to parallel lines (only true of translations).
2. Preserve angles (true of all rigid motions).
3. Preserve length/distance (true of all rigid motions).
CONGRUENCE

Two figures in the plane are congruent if a sequence of rigid motions can be found that make the two figures coincide (or lie exactly on top of each other). We call this sequence of rigid motions a congruence.

PROPERTIES OF RIGID MOTIONS

1. All rigid motions map lines to lines, segments to segments, and rays to rays.
2. All rigid motions preserve distance and angle measurements.
3. Rotation of a line $180^\circ$ about a point not on the line produces a parallel line.
4. Rotation of a line $180^\circ$ about a point that does lie on the line produces the same line.
5. Translation of a line not along the line produces a parallel line.
6. When a point is reflected across a line, the segment connecting the point and its image is perpendicularly bisected by the line.

Examples:

1. In the diagram below, a sequence of rigid motions maps $ABCD$ onto $JKLM$.

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\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure.png}
\end{figure}
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If $m\angle A = 82^\circ$, $m\angle B = 104^\circ$, and $m\angle L = 121^\circ$, the measure of $\angle M$ is

(1) $53^\circ$ \hspace{1cm} (3) $104^\circ$

(2) $82^\circ$ \hspace{1cm} (4) $121^\circ$
2. The graph below shows two congruent triangles, $ABC$ and $A'B'C'$. Which rigid motion would map $\triangle ABC$ onto $\triangle A'B'C'$?

(1) a rotation of 90 degrees counterclockwise about the origin  
(2) a translation of three units to the left and three units up  
(3) a rotation of 180 degrees about the origin  
(4) a reflection over the line $y = x$

3. The vertices of $\triangle PQR$ have coordinates $P(2,3), Q(3,8)$, and $R(7,3)$. Under which transformation of $\triangle PQR$ are distance and angle measure preserved?

(1) $(x,y) \rightarrow (2x, 3y)$  
(3) $(x,y) \rightarrow (2x, y + 3)$  
(2) $(x,y) \rightarrow (x + 2, 3y)$  
(4) $(x,y) \rightarrow (x + 2, y + 3)$
In the graph below, \( \triangle ABC \) has coordinates \( A(-9,2) \), \( B(-6,-6) \), and \( C(-3,-2) \), and \( \triangle RST \) has coordinates \( R(-2,9) \), \( S(5,6) \), and \( T(2,3) \).

Is \( \triangle ABC \) congruent to \( \triangle RST \)? Use the properties of rigid motions to explain your reasoning.
5. Triangle ABC and triangle ADE are graphed on the set of axes below.

Describe a transformation that maps triangle ABC onto triangle ADE.

Explain why this transformation makes triangle ADE similar to triangle ABC.

6. The image of ΔDEF is ΔD'E'F'. Under which transformation will the triangles not be congruent?
   (1) a reflection through the origin
   (2) a reflection over the line y = x
   (3) a dilation with a scale factor of 1 centered at (2,3)
   (4) a dilation with a scale factor of \( \frac{3}{2} \) centered at the origin

7. Which transformation would not always produce an image that would be congruent to the original figure?
   (1) translation
   (2) dilation
   (3) rotation
   (4) reflection
8. As shown in the graph below, the quadrilateral is a rectangle.

Which transformation would \textit{not} map the rectangle onto itself?

(1) a reflection over the $x$-axis
(2) a reflection over the line $x = 4$
(3) a rotation of $180^\circ$ about the origin
(4) a rotation of $180^\circ$ about the point (4,0)

9. A regular decagon is rotated $n$ degrees about its center, carrying the decagon onto itself. The value of $n$ could be

(1) $10^\circ$ \hspace{1cm} (3) $225^\circ$
(2) $150^\circ$ \hspace{1cm} (4) $252^\circ$
10. Quadrilateral \( MATH \) and its image \( M'A'T'H' \) are graphed on the set of axes below.

Describe a sequence of transformations that maps quadrilateral \( MATH \) onto quadrilateral \( M'A'T'H' \).

11. In the diagram below of \( \triangle ABC \) and \( \triangle XYZ \), a sequence of rigid motions maps \( \triangle A \) onto \( \triangle X \),\n\( \triangle C \) onto \( \triangle Z \), and \( \overline{AC} \) onto \( \overline{XZ} \).

Determine and state whether \( \overline{BC} = \overline{YZ} \). Explain why.
Which sequence of transformations maps $\triangle ABC$ onto $\triangle DEF$?

1. a reflection over the x-axis followed by a translation
2. a reflection over the y-axis followed by a translation
3. a rotation of 180° about the origin followed by a translation
4. a counterclockwise rotation of 90° about the origin followed by a translation

In the two distinct acute triangles $ABC$ and $DEF$, $\angle B \equiv \angle E$. Triangles $ABC$ and $DEF$ are congruent when there is a sequence of rigid motions that maps

1. $\angle A$ onto $\angle D$, and $\angle C$ onto $\angle F$
2. $\overline{AC}$ onto $\overline{DF}$, and $\overline{BC}$ onto $\overline{EF}$
3. $\angle C$ onto $\angle F$, and $\overline{BC}$ onto $\overline{EF}$
4. point $A$ onto point $D$, and $\overline{AB}$ onto $\overline{DE}$
Triangle $ABC$ and triangle $DEF$ are drawn below.

If $AB \cong DE$, $AC \cong DF$, and $\angle A \cong \angle D$, write a sequence of transformations that maps triangle $ABC$ onto triangle $DEF$.

Triangle $ABC$ has vertices at $A(-5,2)$, $B(-4,7)$, and $C(-2,7)$, and triangle $DEF$ has vertices at $D(3,2)$, $E(2,7)$, and $F(0,7)$. Graph and label $\triangle ABC$ and $\triangle DEF$ on the set of axes below.

Determine and state the single transformation where $\triangle DEF$ is the image of $\triangle ABC$.

Use your transformation to explain why $\triangle ABC \cong \triangle DEF$.
Identify which sequence of transformations could map pentagon $ABCDE$ onto pentagon $A''B''C''D''E''$, as shown below.

(1) dilation followed by a rotation
(2) translation followed by a rotation
(3) line reflection followed by a translation
(4) line reflection followed by a line reflection

The graph below shows $\triangle ABC$ and its image, $\triangle A''B''C''$.

Describe a sequence of rigid motions which would map $\triangle ABC$ onto $\triangle A''B''C''$. 